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Iterative Dual Rational Krylov and Iterative SVD-Dual Rational Krylov Model Reduction for Switched Linear Systems

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***Abstract*—In this paper, we propose two models reduction algorithms for approximation of large-scale linear switched systems. We present at first the iterative dual rational krylov approach, that construct a union of krylov subspaces. The iterative dual rational Krylov is low in cost, numerical efficient but the stability of reduced system is not always guaranteed. In the second part we present, the iterative SVD-Dual Rational Krylov approach. This method is a combining of two sided-projections, one side is generated by the dual Rational krylov-based model reduction techniques and the other side is generated by the SVD model reduction techniques, while the SVD-side depends on the observability gramian. This method is numerical efficient, minimize the H_∞ Error between the original switched system and reduced one and preserved always the stability of reduced systems. A simulation two examples are considered in order to take a performance study of these proposed approaches.**

I. Introduction

The technological advance means the increase of the complexity of industrial systems. They operate in different environments with changing conditions and characteristics (quickly and brutally). Such as the industries aerospace, automotive, aggro-food, process engineering, chemical process, electrical circuit, power electronic systems, Thermal fluid systems and Mechanical system.... The modeling of these system types usually leads to the production of non-linear complex models of high order. However the dynamic is influenced by both discrete and continuous event which leads to the hybrid dynamical systems which are divided into two broad classes of hybrid systems, the first class is the multi-model system, which assumes that it is always possible to model a complex system with simple models, often linear models, assigning each model an operating area of the system. The second class is the linear piecewise system or called the switched system. This class of models is widely used for the analysis and to control tools for linear systems which are very developed and also because much of the actual process can be represented by models from this class. Recent research on switching systems are mainly focusing on the modeling area, design control law and stability study. However, if we work for the development of the control law, it must first take into consideration the order of the controlled system , because there are several hybrid systems of high order. These laters are difficult to manipulate and the resolution of such models is indeed very demanding in computational resources. However, reduction of switched systems is an important solution for these problems. In this paper , we present the iterative dual rational krylov algorithm and the iterative SVD- dual rational krylov algorithm for switched linear system.
The model reduction problem we are interested in can be stated as follows.
Given a switched linear dynamical system in state space form [1, 2, 3, 4]:

$$\Sigma_q = \begin{cases} \frac{dx(t)}{dt} = A_q x(t) + B_q u(t) \\ y(t) = C_q x(t) + D_q u(t) \end{cases} \tag{1}$$

In which $A_q \in \mathbb{R}^{n\times n}$, $B_q \in \mathbb{R}^{n\times p}$, $C_q \in \mathbb{R}^{p\times n}$, $D_q \in \mathbb{R}^{p\times p}$, $u(t) \in \mathbb{R}^{n\times p}$, $y(t) \in \mathbb{R}^{p\times n}$ and q is a switching signal.
The transfer function of the original system is given by [5, 6]:

$$f_q(s) = C_q(sI_q - A_q)^{-1}B_q + D_q \tag{2}$$

The problem consist in approximating : $A_{r_q} \in \mathbb{R}^{r\times r}$, $B_{r_q} \in \mathbb{R}^{r\times p}$,$C_{r_q} \in \mathbb{R}^{p\times r}$, $D_{r_q} \in \mathbb{R}^{p\times p}$ and $y_r(t) \in \mathbb{R}^{p\times r}$, the matrices of the each reduced subsystem of order r_q, where $r_q \ll n$.
The state space representation of reduction switched dynamical linear systems is as follows [7, 8, 1, 3]:

$$\hat{\Sigma}_q = \begin{cases} \frac{dx_r(t)}{dt} = A_{r_q} x(t) + B_{r_q} u(t) \\ y_r(t) = C_{r_q} x(t) + D_{r_q} u(t) \end{cases} \tag{3}$$

This paper is organized as follows. Section 2, briefly presents an overview of the Lyapunov equations and the H_∞ error. In section 3, the Dual Rational Krylov is presented. Section 4, the Iterative Dual Rational Krylov method for switched linear systems, will be presented with application on the numerical examples. In section 5, we detailed the Iterative SVD-Dual Rational Krylov method for switched linear systems and evaluate by the use of the numerical examples. In section 6, we give a comparison between the Iterative Dual Rational

Krylov method and the Iterative SVD-Dual Rational krylov method. The last section is dedicated to conclude this paper.

II. Basic Tools

A. Lyapunov equations

Let a switched linear stable system as in (1). The solution of this system in the sense of lyapunov is obtained by solving the following two equations for each subsystem [9, 3]:

$$\begin{cases} A_q P_q + P_q A_q^T + B_q B_q^T = 0 \\ A_q^T Q_q + Q_q A_q + C_q^T C_q = 0 \end{cases} \tag{4}$$

The solutions of these two equations are P and Q [10, 9]. $P_q \in \mathbb{R}^{n\times n}$ and $Q_q \in \mathbb{R}^{n\times n}$ are called the reachability and the observability gramians matrices, respectively. Gramians matrices plays an important part in the reduction methods based on singular value decomposition. The relationship between the singular value decomposition and the gramians matrices is as follows [9]:

$$\sigma_{i_q} = \sqrt{\lambda_{i_q}(P_q Q_q)} \tag{5}$$

Where, σ_{i_q} presents the Hankel singular value of Σ_q

B. H_∞ of Dynamical Switched System

In my work, determining the error between the original switched system and reduced one is obtained by using the H_∞ technique knowing that [10, 9]:

$$\|\Sigma_q(jw) - \hat{\Sigma}_q(jw)\|_{H_\infty} \leq 2(\sigma_{(r+1)_q} + ... + \sigma_{n_q}) \tag{6}$$

III. Dual Rational Krylov for Switched Linear System

In this section we briefly recall the details of the Dual Rational Krylov algorithm for computing of two projection matrices V_{r_q} and Z_{r_q} for each subsystem according to switching signal q. Dual Rational Krylov is among the best approaches to reduce the large-scale linear switched systems. It is easy to implemented, numerically stable and avoids the difficulties in the constructing of the two projections matrices. V_{r_q} and Z_{r_q} are constructed column by column during the iteration process using a Gram Schmidt techniques in orthogonalization procedure, such as the condition of biorthogonalithy is satisfied $Z_{r_q}^T V_{r_q} = I_{r_q}$.
Take a switched linear system as a form (1) and assume that a sequence of expansion points $\{s_{1_q}, s_{2_q}, ..., s_{r_q}\}$ is given, with r is the order of reduced subsystem. These expansion points are interspersed. For each expansion point of each subsystem a two column vectors are generated, i.e in the first iteration uses s_{1_q}, the second iteration uses s_{2_q} until $r^{th} iteration$.
The details of the Dual Rational Krylov algorithm for switched linear system can be found in table 1 [11, 9, 12, 13]:
The parameters of the reduced system can be obtained by the congruence transformation:
$A_{r_q} = Z1 A_q V_q,\ B_{r_q} = Z1 B_q,\ C_{r_q} = C_q V_q,\ D_{r_q} = D_q$.
Where, $Z1 = ((Z_q^T V_q)^{(-1)} * Z_q^T)$

TABLE I. DRK-SLS Algorithm

DRK-SLS Algorithm:(input:$I_q, A_q, B_q, C_q, D_q, S_q$;output:$V_{r_q}, Z_{r_q}$)

Switch q
1/*Choose the Initial Interpolation points*/
s_{i_q} for i_q=1 to r_q
2/*Construction of the matrices V_q and Z_q by the Dual rational-Krylov based method knowing that*/
for k_q=1 to r_q
if k_q:=1
$v0_q = ((A_q - s_q * I_q)^{-1} * B_q$
$v0_q = v0_q / norm(v0_q, 'fro')$
$V_q(:,1) = v0_q$
$z0_q = ((A_q - s_q * I_q)^{-T} * C_q^T$
$z0_q = z0_q / norm(z0_q, 'fro')$
$Z_q(:,1) = z0_q$
else
$v_q(:,k) = ((A_q - s_q * I_q)^{-1} * B_q$
$v_q(:,k) = v_q(:,k) - V_q(:,k-1) * V_q(:,k-1)^T * v_q(:,k)$
$V_q(:,k) = v_q(:,k) / norm(v_q(:,k), 'fro')$
$z_q(:,k) = ((A_q - s_q * I_q)^{-T} * C_q^T$
$z_q(:,k) = z_q(:,k) - Z_q(:,k-1) * Z_q(:,k-1)^T * z_q(:,k)$
$Z_q(:,k) = z_q(:,k) / norm(z_q(:,k), 'fro')$
End if
End for
End Switch

A. Numerical example

To evaluate this approach ,we take tow switched linear stable models (FOM of order 1006 and Clamped Beam of order 348) and a switched signal where q=1,2. We present the largest singular value of the frequency response, the distribution poles of the reduced switched system and the absolute error between original subsystem and reduced one. The parameters of States representation of two models are as follows [14, 15, 16]:
Example 1: Switched FOM model of order 1006
$A_1 = diag(\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ with,
$\gamma_1 = \begin{pmatrix} -1 & -100 \\ -100 & -1 \end{pmatrix}$,
$\gamma_2 = \begin{pmatrix} -1 & -200 \\ -200 & -1 \end{pmatrix}$,
$\gamma_3 = \begin{pmatrix} -1 & -400 \\ -400 & -1 \end{pmatrix}$,
$\gamma_4 = diag(-1, ..., -1000)$,
$B_1 = [10 * ones(6,1); ones(1000,1)],\ C_1 = B_1^T,\ D_1 = 0.$
$A_2 = A_1 - 5 * I,\ B_2 = B_1,\ C_2 = C_1,\ D_2 = D_1.$
Example 2: Switched Clamped Beam model of order 348
The Clamped beam model is a theoretical stable model of order 348 contains two subsystems:
The first subsystem is introduced in [14],
The second subsystem is as follows:
$A_2 = A_1 - 5 * 10^{-1} * I,\ B_2 = B_1,\ C_2 = C_1,\ D_2 = 0.$

The figure 1 presents the largest singular value of the frequency response of the original switched linear system (FOM order 1006) and reduced one (order 10) to a frequency range by DRK-SLS method. We can see when a correlation over the entire frequency range shape with a low error rate for low frequency. The figure 2 shows the variation of the singular value of the absolute error between the original switched linear system and the reduced one, we see that the error is small around the low frequency. The distribution poles in the complex plane of each subsystem is depicted in figure 3, all poles are negative real part, then the reduced switched linear system is stable.

The figure 4 presents the largest singular value of the frequency response of the original switched linear system (Beam order 348) and reduced one (order 24) to a frequency range by DRK-SLS method. We can see when a correlation over the entire frequency range shape with a low error rate for low frequency for the second subsystem. However for the first subsystem, I see that the correlation is not good. The figure 5 shows the variation of the singular value of the absolute error between the original switched linear system and the reduced one, we see that the error is small around the low frequency for the second subsystem, which is not the case for the first subsystem. The distribution poles in the complex plane of each subsystem is depicted in figure 6, we note the existence of positive real part poles, then the reduced switched linear system is unstable.

IV. Iterative Dual Rational Krylov for Linear Switched System

In this section we present the proposed method, the iterative dual rational Krylov model reduction for switched linear system, is an extended version of the dual rational krylov method for switched linear system. Iterative dual rational krylov is a connection between the krylov-based reduction method and the interpolation of the expansion points. Given a stable switched linear system as the form (1) and using the eigenvalues criterion in the choice of the interpolation points [12, 1]. However, this method generated two Krylov subspaces V_{r_q} and Z_{r_q} for each subsystem, the generation of the two krylov subspaces is performed iteratively until the satisfaction of the stopping criterion $((s_{(i+1)_q} - s_{i_q})/s_{(i+1)_q})$ and guarantees the biorthogonalithy condition of the two krylov subspaces for each subsystem (i.e. $Z_{r_q}^T V_{r_q} = I_{r_q}$ where r_q is the order of reduced system) [17, 11, 18, 19]. Theorem 1 summarizes this result:

theorem Take a switched linear system as a form (1) and the interpolation point $\{s_{i_q}\}$ for $i_q = 1,\ldots,r_q$. Let $V_{r_q} \in \mathbb{R}^{n\times r}$ and $Z_{r_q} \in \mathbb{R}^{n\times r}$ be obtained as follows [6]:

$$\begin{cases} V_{r_q} = Span(s_{1_q}I - A_q)^{-1}B_q,\ldots,(s_{r_q}I - A_q)^{-1}B_q \\ Z_{r_q} = Span(s_{1_q}I - A_q)^{-T}C_q^T,\ldots,(s_{r_q}I - A_q)^{-T}C_q^T \end{cases} \quad (7)$$

With $Z_{r_q}^T V_{r_q} = I_{r_q}$.
The transfer function $f_{r_q}(s)$ of the reduced switched system (2) is matched with the transfer function $f_q(s)$ of original switched linear system in (1):

$$f_q(s_{i_q}) = f_{r_q}(s_{i_q}) \quad for \quad i_q = 1,\ldots,r_q \quad and \quad s_{i_q} = -\lambda_{i_q}(A_{r_q}) \quad (8)$$

Where, λ_{i_q} is the eigenvalues of A_{r_q} The details of the Iterative Dual Rational Krylov algorithm for switched linear system (IDRK-SLS) can be found in table 2 [17, 11, 18]:
The main steps of this method are:
Step 1: In the first step we choose the interpolation points for each sub-system by the use of the eigenvalues criterion [10].
Step 2: Compute the V_{r_q} and Z_{r_q} bases with Dual Rational Krylov knowing that the orthogonality condition is satisfied

TABLE II. IDRK-SLS Algorithm

IDRK-SLS Algorithm:(input:$I_q, A_q, B_q, C_q, D_q, S_q, tol$; output:$V_{r_q}, Z_{r_q}$)
Switch q 1/*Choose the Initial Interpolation points*/ s_{i_q} for i=1 to r 2/*Construction of the matrices V_q and Z_q by the Dual rational-Krylov based method knowing that*/ (a):$V_{r_q} = Span(s_{1_q}I_q - A_q)^{-1}B_q,\ldots,(s_{r_q}I_q - A_q)^{-1}B_q$ (b):$Z_{r_q} = Span(s_{1_q}I_q - A_q)^{-T}C_q^T,\ldots,(s_{r_q}I_q - A_q)^{-T}C_q^T$ With $Z_{r_q}^T V_{r_q} = I_{r_q}$, where $Z_{r_q} = (Z_{r_q}^T * V_{r_q})^{-1}Z_{r_q}^T$ 3/*While (the relative change in s_i : $((s_{i+1} - s_i)/s_i) \geq tol$ (a):$A_{r_q} = Z_{r_q}A_qV_{r_q}$ (b):$s_{i_q} = -\lambda_{i_q}(A_{r_q})$ for $i = 1 : r_q$ (c):Construction of the matrices V_{r_q} and Z_{r_q} by the rational-Krylov based method knowing that: (d):$V_{r_q} = Span(s_{1_q}I_q - A_q)^{-1}B_q,\ldots,(s_{r_q}I_q - A_q)^{-1}B_q$ (e):$Z_{r_q} = Span(s_{1_q}I_q - A_q)^{-T}C_q^T,\ldots,(s_{r_q}I_q - A_q)^{-T}C_q^T$ With $Z_{r_q}^T V_{r_q} = I_{r_q}$, where $Z_{r_q} = (Z_{r_q}^T * V_{r_q})^{-1}Z_{r_q}^T$ (4)/*parameters of reduced model*/ $A_{r_q} = Z_{r_q}A_qV_{r_q}$, $B_{r_q} = Z_{r_q}B_q$, $C_{r_q} = C_qV_q$, $D_{r_q} = D_q$ **End Switch**

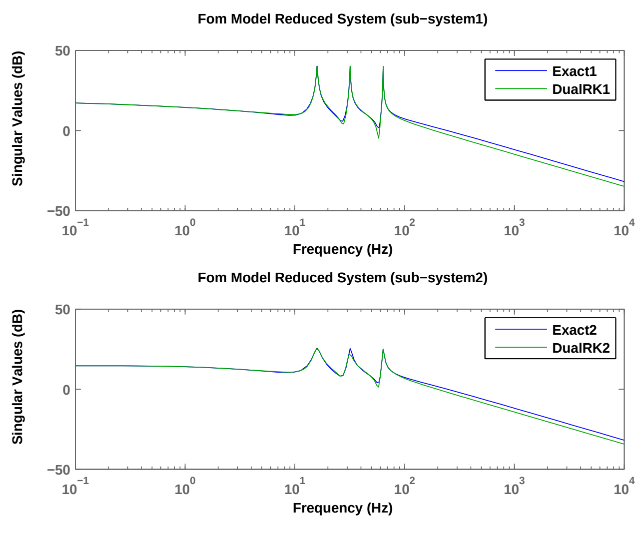

Fig. 1. Largest singular value of the frequency response of the original switched system of order (1006) and reduced one of order (10) to a frequency range with DRK-SLS method

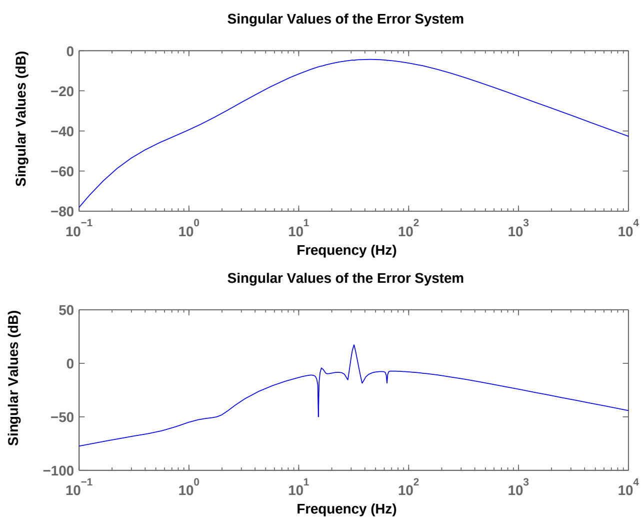

Fig. 2. Absolute error system between original switched system of order (1006) and the reduced one of order (10) with DRK-SLS method

$((Z_{r_q}^T * V_{r_q})^{-1} Z_{r_q}^T) V_{r_q} = I_{r_q})$.
Step 3: Calculate the reduced states matrices A_{r_q} and the corresponding eigenvalues. Using the mirror of these eigenvalues as interpolation points and recalculate the new bases V_{r_q} and Z_{r_q} applying again the Dual Rational Krylov method. Repeat these instructions until the satisfaction of a stopping criterion in the expansion frequency.

A. *Numerical example*

To evaluate this approach ,we take the same models used previously, with the same switching signal. We fix the largest singular value of the frequency response of each original subsystem and the reduced one, we present the variation of absolute error between each original subsystem and the reduced one and we give the poles distribution of the reduced subsystem [14, 15].
The figure 7 presents the largest singular value of the frequency response of the original switched linear system (order 1006) and reduced one (order 10) to a frequency range by IDRK-SLS method. We can see when a correlation over the entire frequency range shape with a low error rate. The figure 8 shows the variation of the singular value of the absolute error between the original switched linear system and the reduced one, we see that the error is small over the entire frequency range. The distribution poles in the complex plane of each subsystem is depicts in figure 9, all poles are negative real part, then the reduced switched linear system are stable.

The figure 10 presents the largest singular value of the frequency response of the original switched linear system (order 348) and reduced one (order 24) to a frequency range by IDRK-SLS method. We can see when a good correlation between the original switched linear system and reduced one over the entire low frequency range shape with a low error rate. The figure 11 shows the variation of the singular value of the absolute error between the original switched linear system and the reduced one, we see that the error is small over the entire low frequency range. The distribution poles in the complex plane of each subsystem is depicts in figure 12, we note the existence of positive real part poles, then the reduced switched linear system is unstable.
We note that the results obtained by this method is better than that obtained by the previous method, but this method does not guarantee the stability of reduced system. To solve this problem we propose in the next section a new method that minimizes the H_∞ error between the original switched linear system and the reduced one and guarantee the stability of reduced switched system.

V. Iterative SVD-Dual Rational Krylov for Switched Linear System

While IDRK-SLS algorithm do not always guarantee stability of the each reduced subsystem, Iterative SVD-Dual Rational Krylov algorithm for linear switched system gives a reduced model with guaranteed stability and minimize the error between the original system and reduced one for each subsystem. Hence, Iterative SVD-Dual Rational Krylov algorithm for linear switched system combines the advantages of the dual rational Krylov based method and the singular value decomposition based method, the use of SVD provide stability for reduced system. This method can generate two matrices, one matrix generated by the Dual Rational Krylov method (V_{r_q}) depends on the observability gramian and the other generated by the singular value decomposition (Z_{r_q}). The two matrices Z_{r_q} and V_{r_q} satisfy the following orthogonality relation [10, 20, 18, 21]:

$$Z_{r_q}^T V_{r_q} = I_r \tag{9}$$

theorem Take a stable switched linear system Σ_q with the transfer functions $f_q(s)$ as in (1) and fix the the interpolation points s_{i_q}, Let Σ_{r_q} be an *rth* reduced sub-systems with transfer functions $f_{r_q}(s)$ having fixed stable reduced poles $\lambda_{1_q}, ..., \lambda_{r_q}$. Then the error between each original subsystem and reduced one is minimized if and only if:

$$f_q(s) = f_{r_q}(s) \quad for \quad s = -\lambda_{1_q}, ..., -\lambda_{r_q} \tag{10}$$

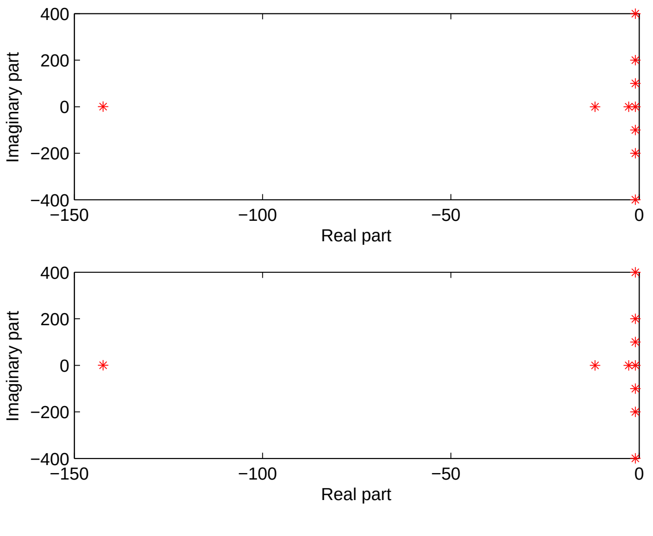

Fig. 3. Poles Distribution of FOM reduced switched system (order 10) with DRK-SLS method

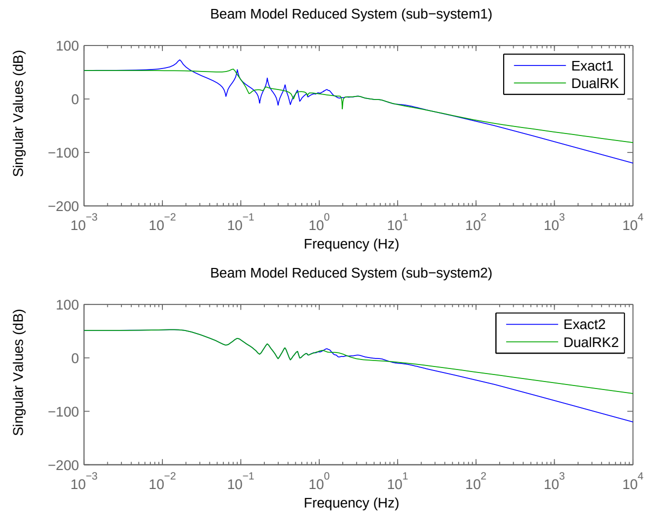

Fig. 4. Largest singular value of the frequency response of the original switched system (Beam of order 348)and reduced one of order (24) to a frequency range with DRK-SLS method

The reduced order model is defined by these relationships:

$$A_{r_q} := Z_{r_q}^T A_q V_{r_q}, \quad B_{r_q} := Z_{r_q}^T B_q, \quad C_{r_q} := C_q V_{r_q}; \quad D_{r_q} := D_q. \tag{11}$$

The different steps of the Iterative SVD-Dual Rational Krylov Algorithm for switched linear system can be found in table 3 [10, 22, 18]:

The main steps of Iterative SVD-Dual Rational Krylov algorithm for switched linear system are:

TABLE III. ITERATIVE SVDDRK-SLS ALGORITHM

Iterative SVDDRK-SLS Algorithm:(input:$I_q, A_q, B_q, C_q, D_q, S_q, tol$; output:$V_{rq}, Z_{rq}$)

Switch q
1/*Choose the Initial Interpolation points*/
s_{i_q} for i_q=1 to r_q
2/*Construction of the matrices V_q by the Dual Rational-Krylov based method knowing that*/
(a):$V_{r_q} = Span(s_{1_q} I_q - A_q)^{-1} B_q, ..., (s_{r_q} I_q - A_q)^{-1} B_q$
With $V_{r_q}^T V_{r_q} = I_{rq}$
3/*Calculate the garamian matrix of observability for each subsystem*/
(a)$g_{o_q} = \int_0^\infty e^{tA_q^T} C_q^T C_q e^{tA} dt$
4/*Construction of the matrix Z_q by the SVD based method knowing that*/
(b):$Z_{r_q} = Q_q V_{r_q} (V_{r_q}^T)$
5/*While (the relative change in $s_i : ((s_{i+1} - s_i)/s_i) \geq tol$
(a):$A_{r_q} = Z_{r_q}^T A_q V_{r_q}$
(b):$s_{i_q} = -\lambda_{i_q}(A_{r_q})$ for i_q=1:r_q
(c):Construction of the matrix V_q by the rational-Krylov based method knowing that:
(d):$V_{r_q} = Span(s_{1_q} I_q - A_q)^{-1} B_q, ..., (s_{r_q} I_q - A_q)^{-1} B_q$
With $Z_{r_q}^T V_{r_q} = I_{rq}$
(e):*Construction of the matrix Z_{r_q} by the SVD based method knowing that*/
$Z_{r_q} = Q_q V_{r_q} (V_{r_q}^T)$
5/*Generate real V_{r_q} and Z_{r_q} for complex interpolation point*/
if there exist any s_{i_q} is not a real number
then $V_{r_q}(real) = real(V_{r_q})$, $V_{r_q}(imaginairy) = imag(V_{r_q})$,
$Z_{r_q}(real) = real(Z_{r_q})$, $Z_{r_q}(imaginairy) = imag(Z_{r_q})$,
$[V_{r_q}, rr_q] = QR[V_{r_q}(real), V_{r_q}(imaginairy)]$,
$[Z_{r_q}, rr_q] = QR[Z_{r_q}(real), Z_{r_q}(imaginairy)]$,
end if
6/*Parameters of reduced model*/
$A_{r_q} = Z_{r_q}^T A_q V_{r_q}, \quad B_{r_q} = Z_{r_q}^T B_q, \quad C_{r_q} = C_q V_{r_q}, \quad D_{r_q} = D_q$
End Switch

Step 1: In the first step we choose the interpolation points for each subsystem by the use of criterion poles, the number of interpolation points must be equal to the order of reduced subsystem.
Step 2: Use the based method of dual rational Krylov for constructing the V_{r_q} basis knowing that the orthogonality condition is satisfied $(V_{r_q}^T V_{r_q} = I_{r_q})$.
Step 3: Calculate the gramian matrix of observability g_{o_q} for each subsystem.
Step 4: Construct the matrix Z_{r_q} using the observability matrix gramian and the matrix V_{r_q}.
Step 5: Generate the real orthogonal matrices V_{r_q} using the reduced *QR* factorization if there exists any complex interpolation points.
Step 6: Calculate the reduced matrices states A_{r_q} and the corresponding eigenvalues. We determine the eigenvalues of these matrices to re-initialize the interpolation points of each subsystem. Then, we calculate again the orthonormal basis V_{r_q} and the matrix Z_{r_q}. Repeat these instructions until the satisfaction of a stoping criterion in the interpolation points. The stoping criterion is a tolerance which was set at the beginning, that denote the relative change between two successive interpolation points.
Step 7: The reduced order model is defined as:
$A_{r_q} = Z_{r_q}^T A_q V_{r_q}, \;\; B_{r_q} = Z_{r_q}^T B_q, \;\; C_{r_q} = C_q V_{r_q}, \;\; D_{r_q} = D_q.$

A. Numerical example

To evaluate this approach we take the same model used previously, with the same switching signal. Given the largest singular value of the frequency response of each original subsystem and the reduced one, we present the variation of absolute error between each original subsystem and the reduced one and we give the poles distribution of the reduced subsystem [14, 15]. The figure 13 presents the largest singular value of the frequency response of the original switched linear system (order 1006) and reduced one (order 10) to a frequency range by Iterative SVDDRK-SLS method. We can see when a

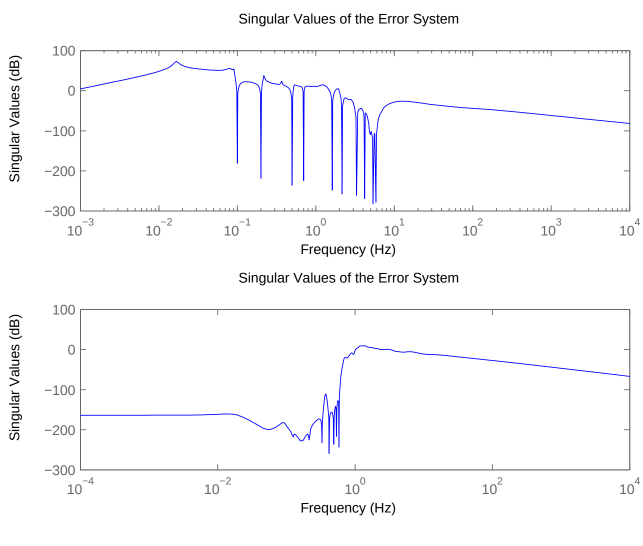

Fig. 5. Absolute error system between original switched system (Beam of order (348)) and the reduced one of order (24) with DRK-SLS method

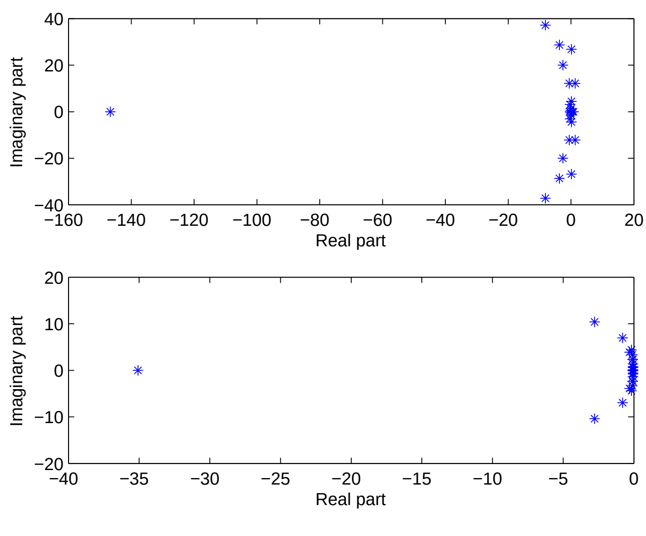

Fig. 6. Poles Distribution of Beam reduced switched system (order 24) with DRK-SLS method

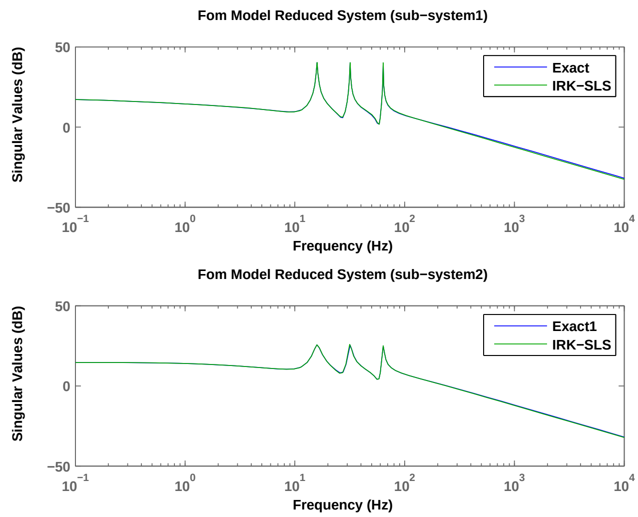

Fig. 7. Largest singular value of the frequency response of the original switched system of order (1006) and reduced one of order (10) to a frequency range with IDRK-SLS method

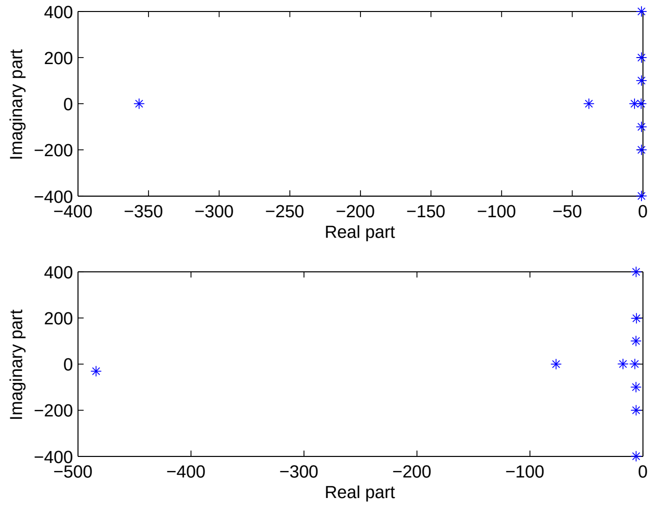

Fig. 9. Poles Distribution of FOM reduced switched system (order 10) with IDRK-SLS method

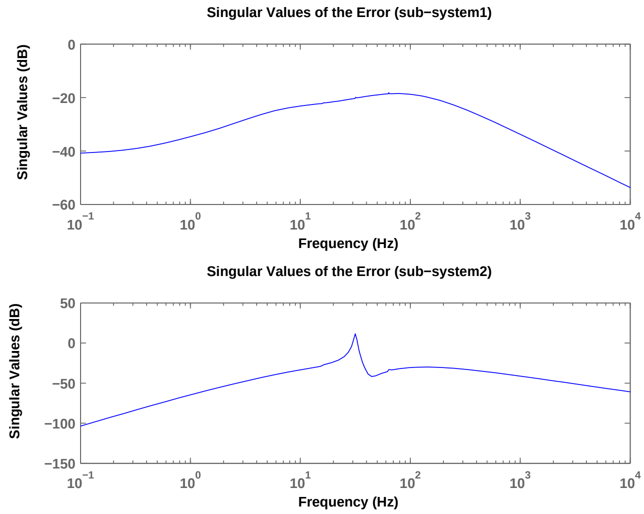

Fig. 8. Absolute error system between original switched system (FOM of order 1006) and the reduced one of order (10) with IDRK-SLS method

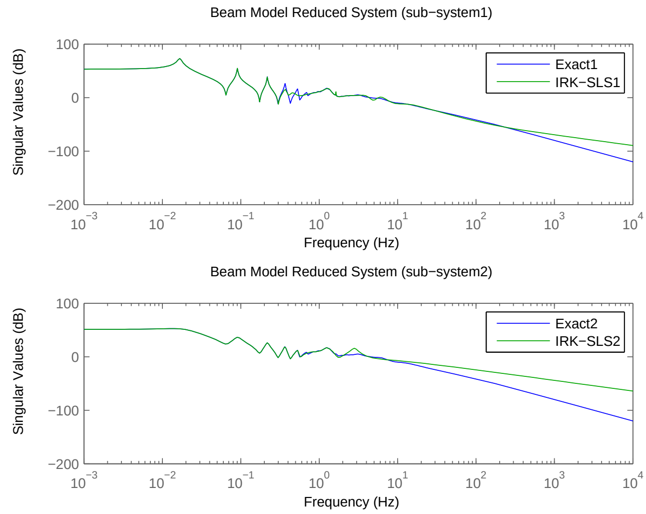

Fig. 10. Largest singular value of the frequency response of the original switched system (Beam of order 348) and reduced one of order (24) to a frequency range with DRK-SLS method

correlation over the entire frequency range shape with a low error rate. The figure 14 shows the variation of the singular value of the absolute error between the original switched linear system and the reduced one, we see that the error is inconsiderable over the entire frequency range. The distribution poles in the complex plane of each subsystems is depicted in figure 15, all poles are negative real part, then the sub-systems is stable.

The figure 16 presents the largest singular value of the frequency response of the original switched linear system (order 348) and reduced one (order 24) to a frequency range by Iterative SVDDRK-SLS method. We can see when a correlation over the entire frequency range shape with a low error rate. The figure 17 shows the variation of the singular value of the absolute error between the original switched linear system and the reduced one, we see that the error is inconsiderable over the entire frequency range. The figure 18 present the distribution poles in the complexes plane of each sub-systems, all poles are negative real part, then the reduced switched linear system is stable.

VI. Comparative Study

In this section, we compare the Iterative Dual Rational Krylov method with Iterative SVD-Dual Rational Krylov method for switched linear system. We present in table 4, the H_∞ error, the CPU-time and the tolerance of each model. We see from the table , that the iterative SVDDRK-SLS has a better results compare to IRK-SLS.

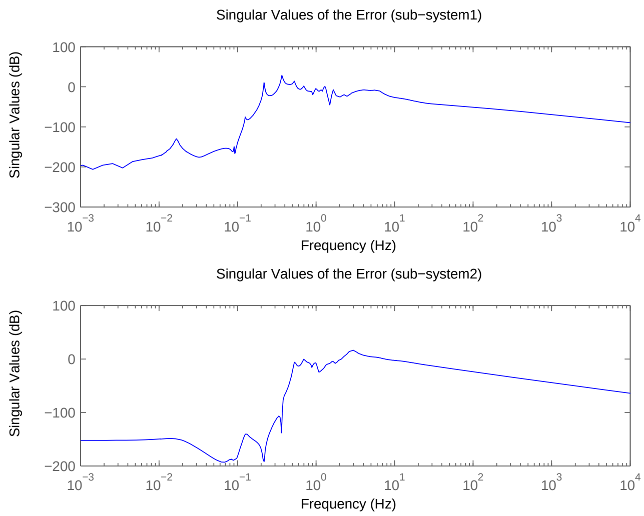

Fig. 11. Absolute error system between original switched system (Beam of order 348) and the reduced one of order (24) with DRK-SLS method

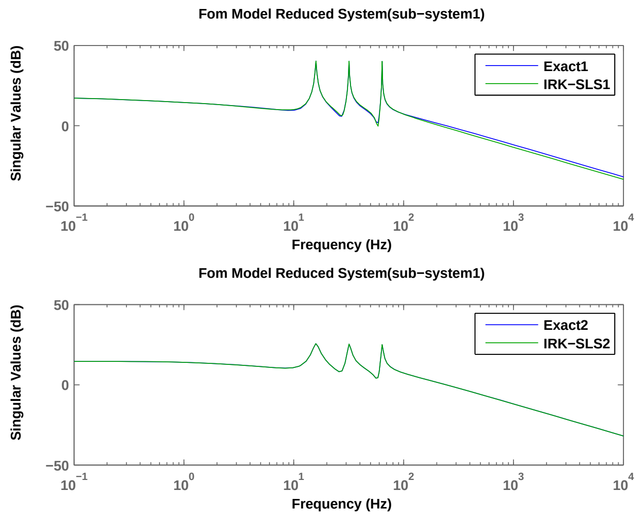

Fig. 13. Largest singular value of the frequency response of the original switched system of order (1006) and reduced one of order (10) to a frequency range with Iterative SVDDRK-SLS method

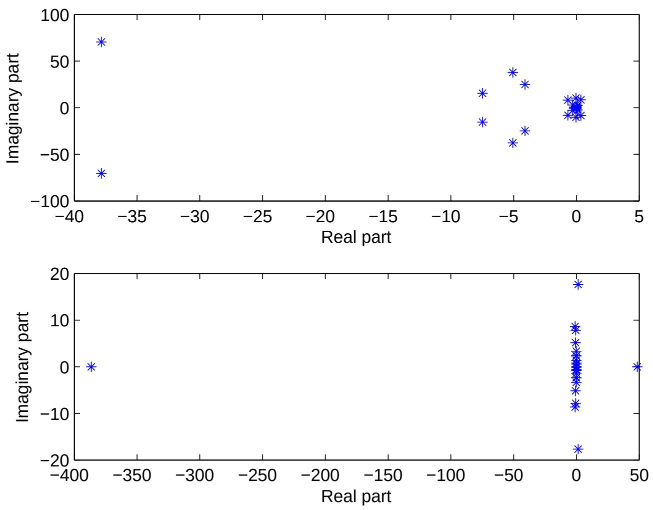

Fig. 12. Poles Distribution of Beam reduced switched system (order 24) with DRK-SLS method

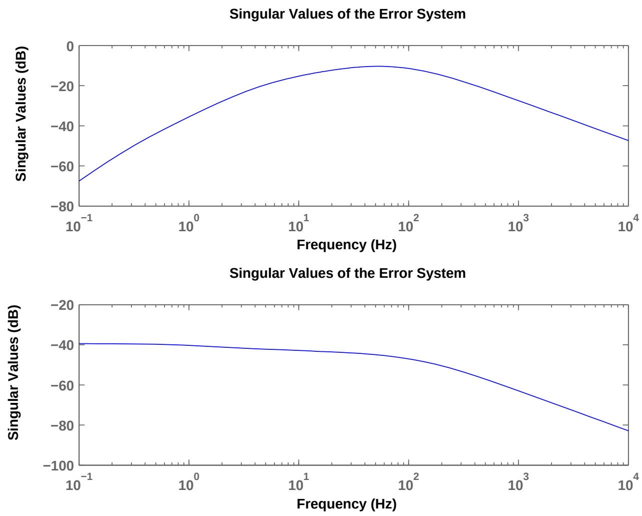

Fig. 14. Absolute error system between original switched system of order (1006) and the reduced one of order (10) with Iterative SVDDRK-SLS method

TABLE IV. H_∞ ERROR, CPU-TIME AND THE TOLERANCE OF ITERATIVE DRK-SLS AND ITERATIVE SVDDRK-SLS

Models	Methods	H_∞ Error	CPU-Time	Tol
FOM 1006	Iterative DRK-SLS	$6.207 * 10^{-9}$	127s	10^{-3}
FOM 1006	Iterative SVDDRK-SLS	$1.877 * 10^{-9}$	110s	10^{-9}
Beam 348	Iterative DRK-SLS	$1.545 * 10^{-8}$	30s	$5 * 10^{-3}$
Beam 348	Iterative SVDDRK-SLS	$1.081 * 10^{-8}$	55s	$2 * 10^{-2}$

VII. CONCLUSION

In this paper we have proposed two methods for reduction of linear switched systems. We present at first the iterative dual rational krylov method based on generation of krylov subspaces. This method have low cost, but the stability of reduced system not always guaranteed. In the second part, we present the iterative SVD-Dual rational krylov based on the SVD and krylov subspace techniques in generating of the projection matrices V_{r_q} and Z_{r_q} for each subsystem. This method is numerically efficient, guaranteed the stability of each reduced subsystems. To evaluate the accuracy and efficient of these methods, we present a numerical examples.

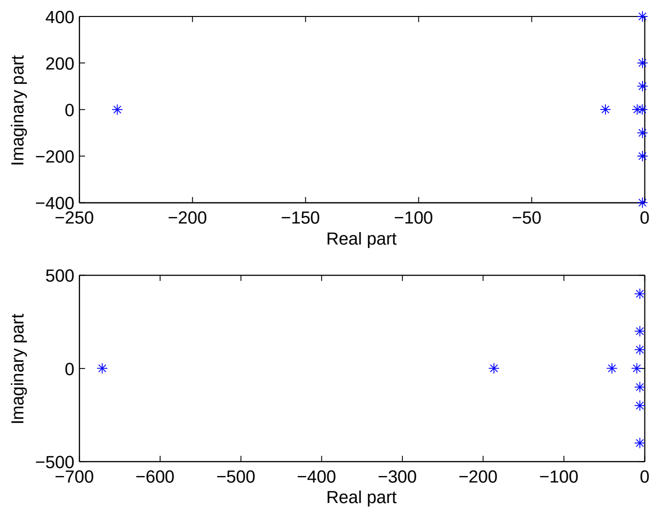

Fig. 15. Poles Distribution of FOM reduced switched system (order 10) with Iterative SVDDRK-SLS method

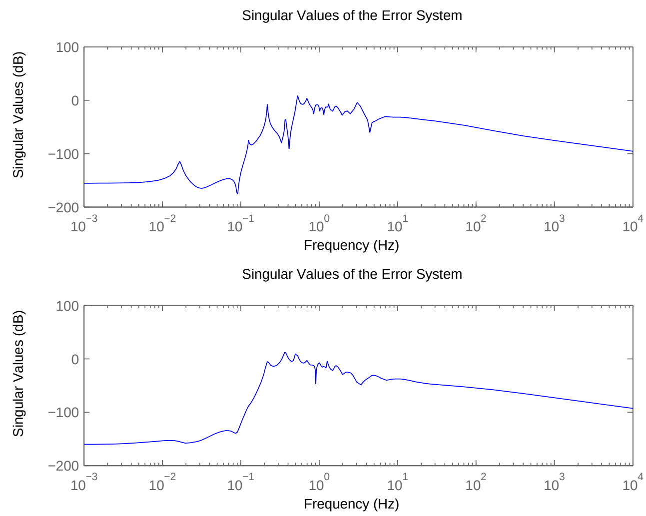

Fig. 17. Absolute error system between original switched system (Beam of order 348) and the reduced one of order (24) with DRK-SLS method

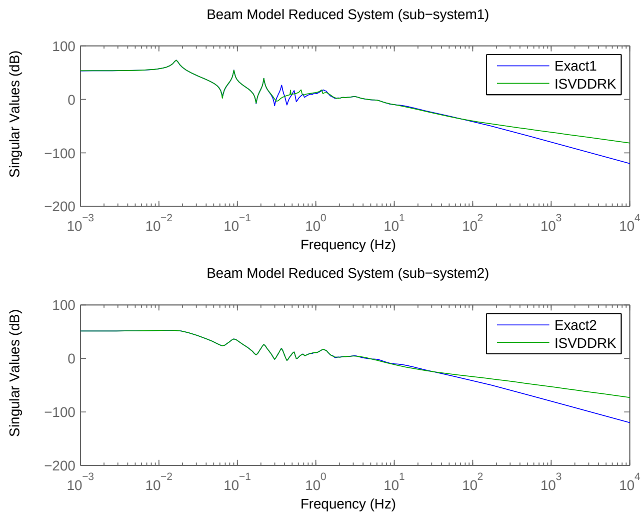

Fig. 16. Largest singular value of the frequency response of the original switched system (Beam of order 348) and reduced one of order (24) to a frequency range with DRK-SLS method

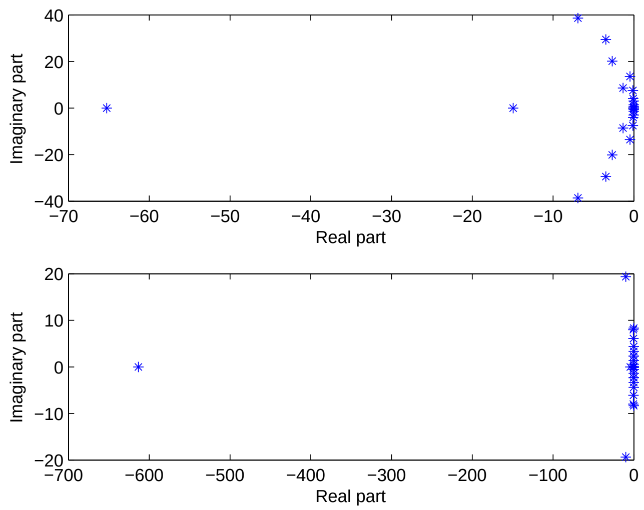

Fig. 18. Poles Distribution of Beam reduced switched system (order 24) with DRK-SLS method

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